

Spheres & Prisms Solutions

1. It would be easier if you visualize this problem.

As sphere is inscribed in cube then the edges of the cube equal to the diameter of a sphere --
 $> \text{Diameter} = 10$.

Next, diagonal of a cube equals to $\text{Diagonal} = \sqrt{10^2 + 10^2 + 10^2} = 10\sqrt{3}$.

Now half of (Diagonal minus Diameter) is a gap between the vertex of a cube and the surface of the sphere, which will be the shortest

$$\text{distance: } x = \frac{\text{Diagonal} - \text{Diameter}}{2} = \frac{10\sqrt{3} - 10}{2} = 5(\sqrt{3} - 1)$$

Answer: D.

2. The surface area of a sphere of radius 0.15 cm is $4\pi r^2 = 4\pi * 0.15^2$;
 The surface area of a sphere of radius 0.3 = 2*0.15 cm
 is $4\pi R^2 = 4\pi * 2^2 * 0.15^2 = 4 * (4\pi * 0.15^2)$.

Since the surface area of the bigger sphere is 4 times the surface area of the smaller sphere and since the weight of a sphere is directly proportional to surface area, then the weight of the bigger sphere is 4 times the weight of the smaller sphere, so $4*8=32$ grams.

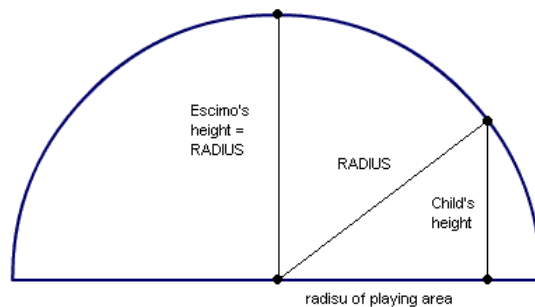
Answer: B.

3. Given: $\text{Volume} = \frac{4}{3}\pi r^3 = 972\pi \rightarrow r = 9$;

So, the surface area would be $4\pi r^2 = 324\pi$.

Answer: D.

4. Look at the diagram below:



Now, the RADIUS of the igloo equals to the height of the Eskimo, so $R = 65$. As the child can play over an area of 9,856 square units then the radius of this playing area

is: $playing\ area = \pi r^2 = 9,856 \rightarrow r^2 = \frac{9,856}{\pi} \rightarrow r \approx 56$. Thus the child's height will be $H = \sqrt{R^2 - r^2} = \sqrt{65^2 - 56^2} = 33$.

Answer: B.

5. There is absolutely no need to actually calculate the height QR. Just notice that the height of a pole (given) and the radius of the pole completely defines (fixes) it. The same way the radius of a sphere completely defines (fixes) it. So, only if we have defined (fixed) pole and defined (fixed) sphere we can be able to say how much below the top of the pole the sphere goes, and we'll be able to calculate QR.

Both statements together provide us with the info needed: the radius of the pole and the radius of the sphere. Hence when taken together statements are sufficient.

Answer: C.

P.S. When dealing with DS problems try to avoid calculations as much as possible. Remember DS problems do not ask you to solve, but rather to determine if you are ABLE to solve and in many cases you can determine that a statement is sufficient without working out all of the math.

6. Say the dimensions of the rectangular solid are a, b and c. It's volume is abc.

ab = 91;
ac = 39,
bc = 21.

Multiply these three: $(abc)^2 = 91 \cdot 39 \cdot 21 = (7 \cdot 13) \cdot (3 \cdot 13) \cdot (3 \cdot 7) = 3^2 \cdot 7^2 \cdot 13^2 \dots$
 $> abc = 3 \cdot 7 \cdot 13 = 273$.

Answer: C.